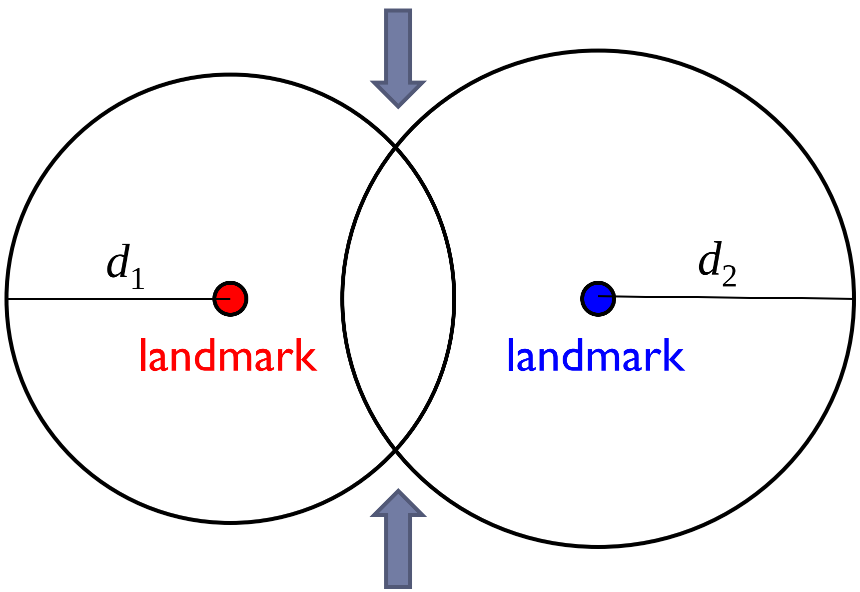
Day 29

Geometric Dilution of Precision

Trilateration

- given two landmarks with known locations we can compute the two possible locations of the robot
  - additional information will tell us which of the two locations (heading information, landmarks are on a wall, etc.)

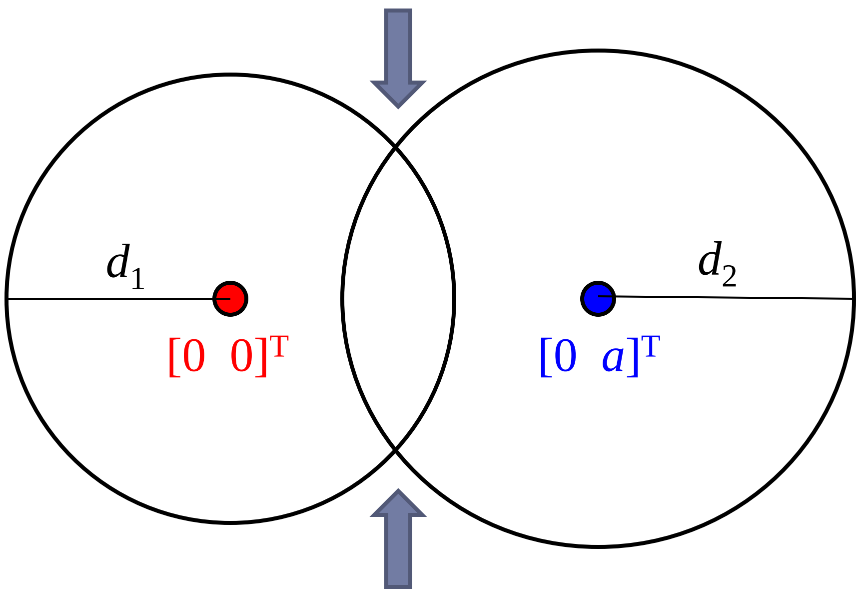


Trilateration

- assume
  - red landmark is located at the origin
  - blue landmark is located a units along the x axis
- then the robot is located at:

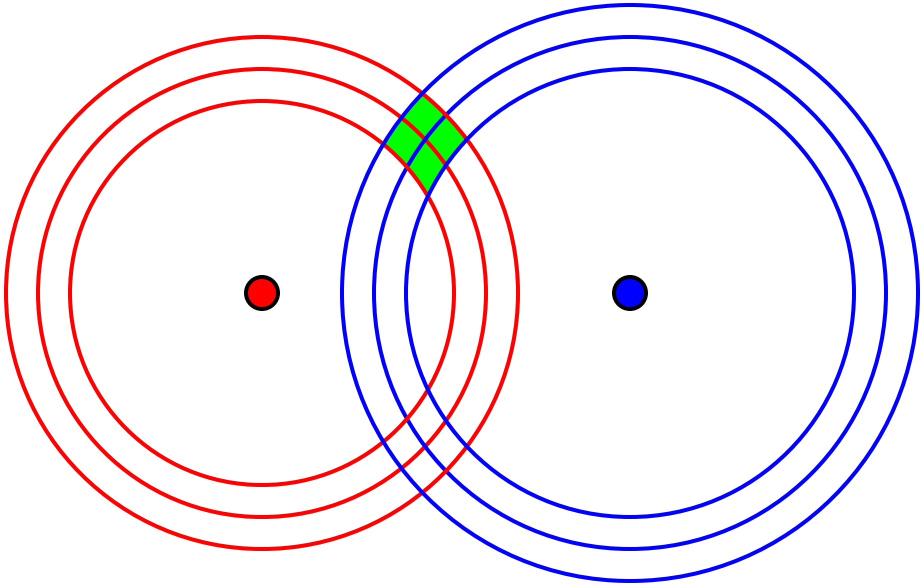
$$x = \frac{a^2 + d_1^2 - d_2^2}{2a}$$

$$y = \pm\left(d_1^2 - x^2\right)^{1/2}$$



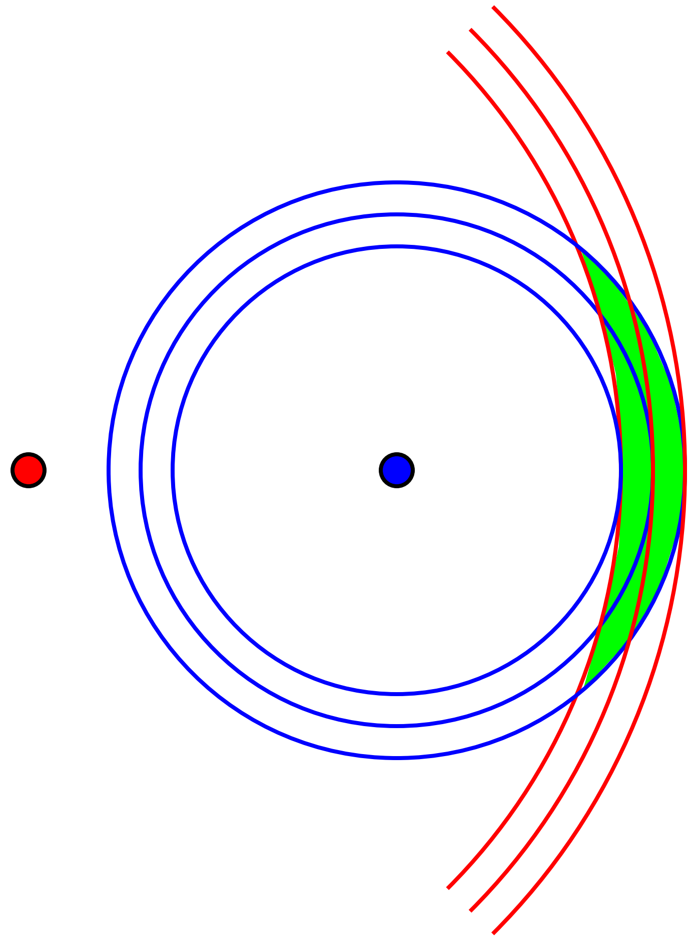
Trilateration

- if the distance measurements are noisy then there will be some uncertainty in the location of the robot



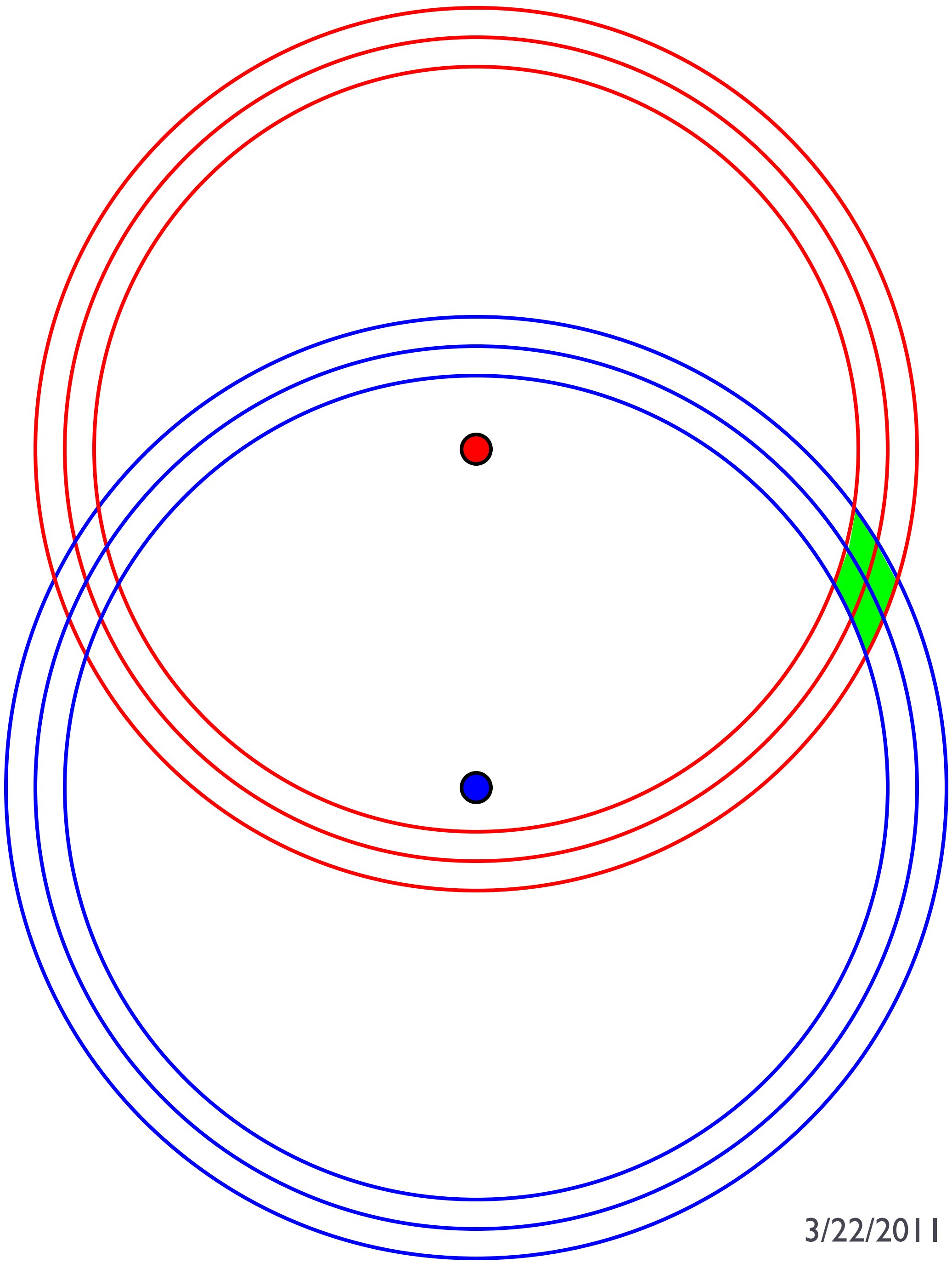
Trilateration

- notice that the uncertainty changes depending on where the robot is relative to the landmarks
- uncertainty grows quickly if the robot is in line with the landmarks



Trilateration

- uncertainty grows as the robot moves farther away from the landmarks
  - but not as dramatically as the previously slide



Geometric Dilution of Precision

- it many cases it might be nice to know how the uncertainty changes as a function of the robot position
  - i.e., how much variation is there in the estimated position for some amount of variation in the distance measurements
    - called the geometric dilution of precision

$$\mathrm{GDOP} = \frac{\Delta \mathbf{X}}{\Delta \mathbf{S}}$$

    - where $\Delta \mathbf{X}$ is the variation in the estimated pose (in this case, position) and $\Delta \mathbf{S}$ is the variation in the sensor readings (in this case, distance)

Geometric Dilution of Precision

- if we take the limit as $\Delta\mathbf{S}\rightarrow\mathbf{0}$ then the GDOP is equal to the Jacobian of the measurement equation

$$x = \frac{a^2 + d_1^2 - d_2^2}{2a}$$

*positive root only $\quad y = \left(d_1^2 - x^2\right)^{1/2}$

$$J = \begin{bmatrix} \dfrac{\partial x}{\partial d_1} & \dfrac{\partial x}{\partial d_2} \\ \dfrac{\partial y}{\partial d_1} & \dfrac{\partial y}{\partial d_1} \end{bmatrix}$$

$$= \begin{bmatrix} d_1/a & -d_2/a \\ d_1(a-x)/(ay) & xd_2/(ay) \end{bmatrix}$$

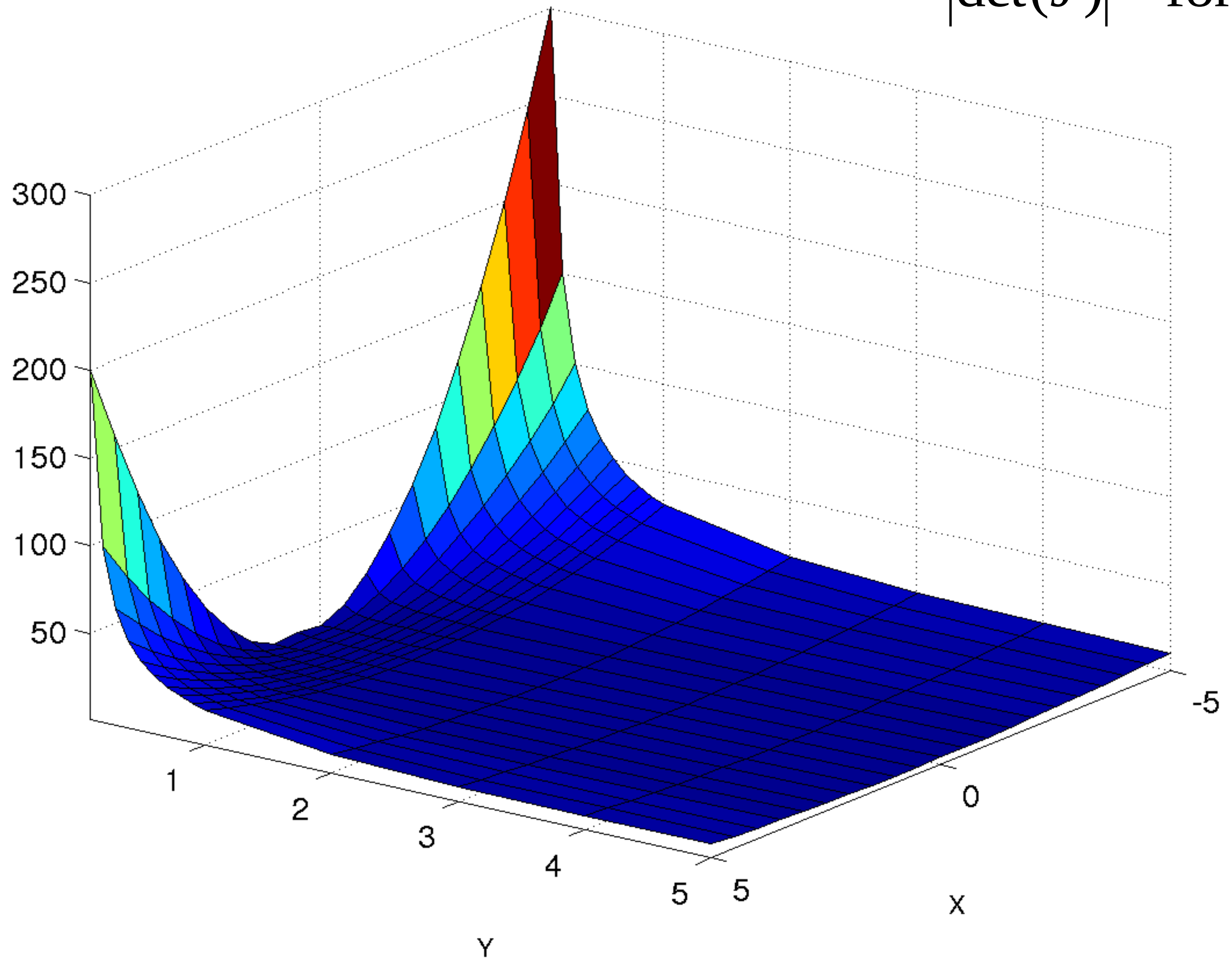
*different than textbook

Geometric Dilution of Precision

- if the Jacobian is a square matrix, then we can examine its determinant
  - the absolute value of the determinant tells you something about how much x and y will change
    - the textbook calls the absolute value of the determinant the magnitude of J

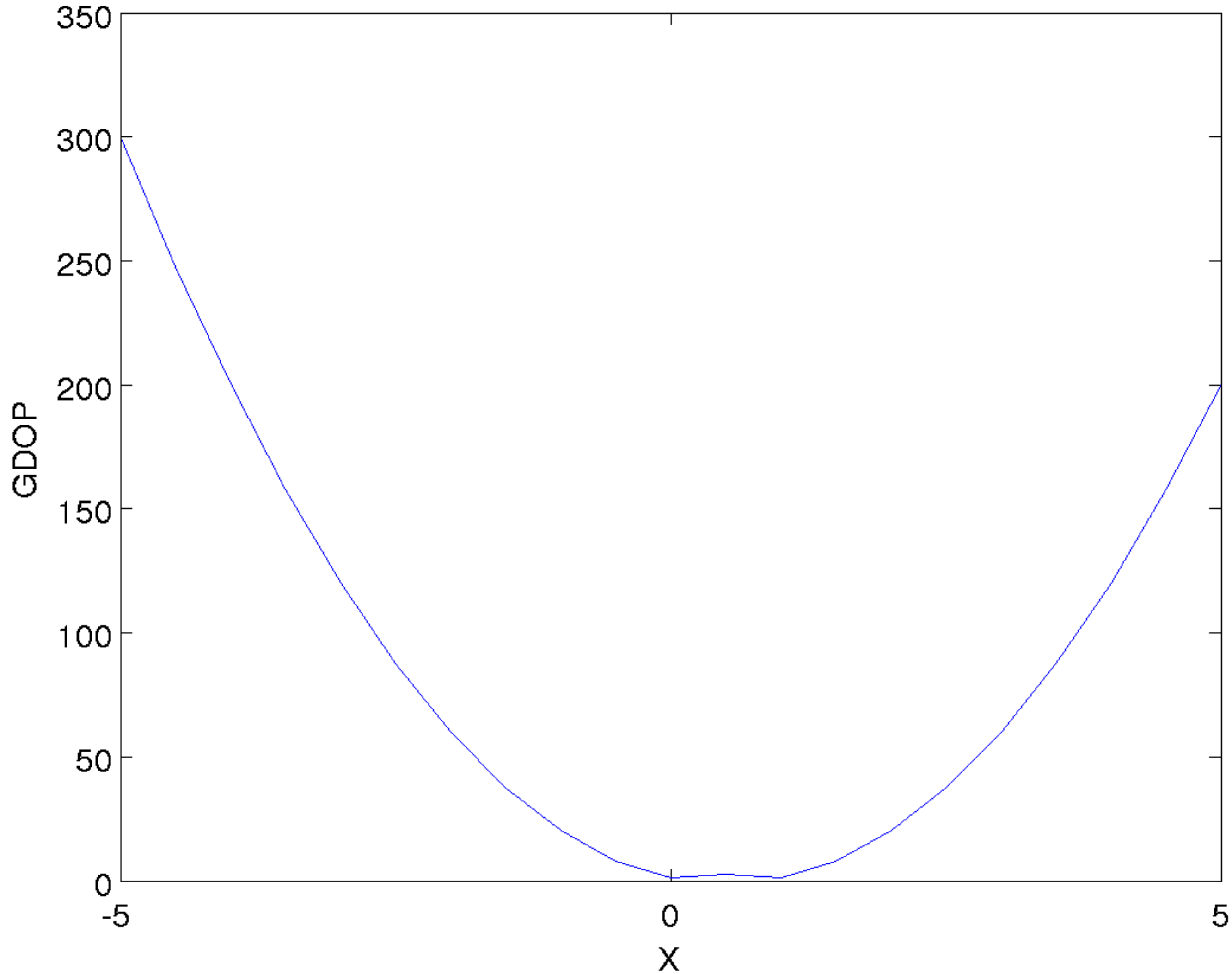
Geometric Dilution of Precision

$$\left|\det(J)\right| \quad \text{for} \quad a = 1$$



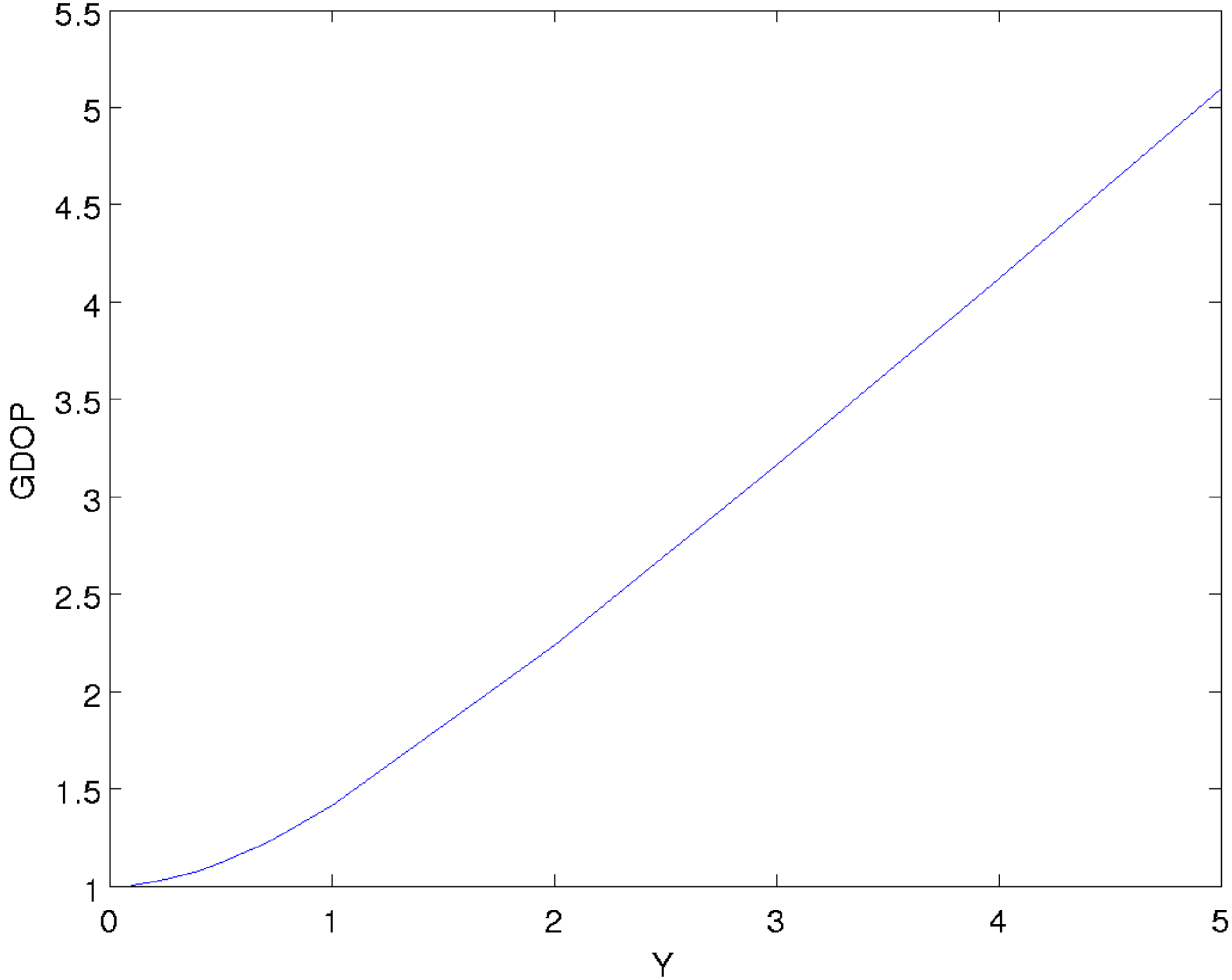
Geometric Dilution of Precision

- the GDOP is very sensitive to changes in x along the y axis



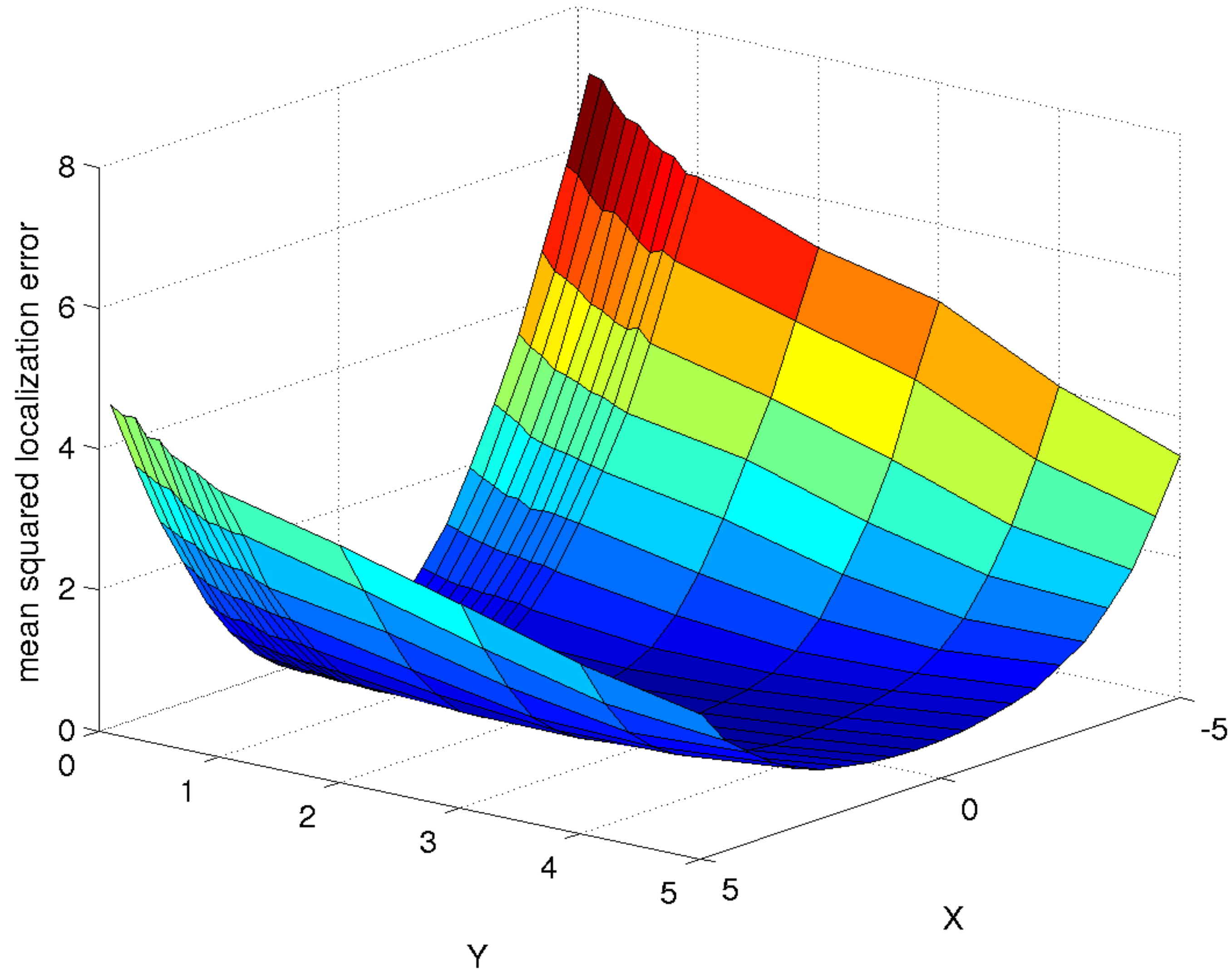
Geometric Dilution of Precision

- the GDOP is less sensitive to changes in *y* along the *x* axis



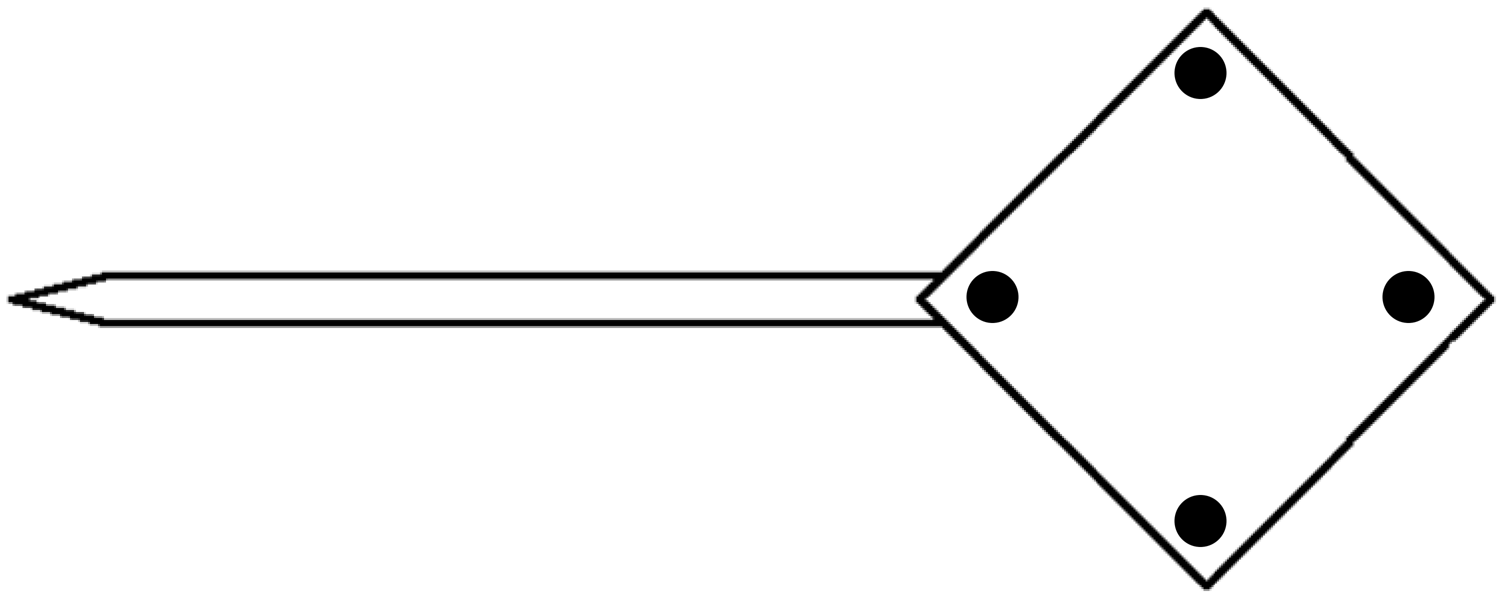
Geometric Dilution of Precision

- simulated localization error



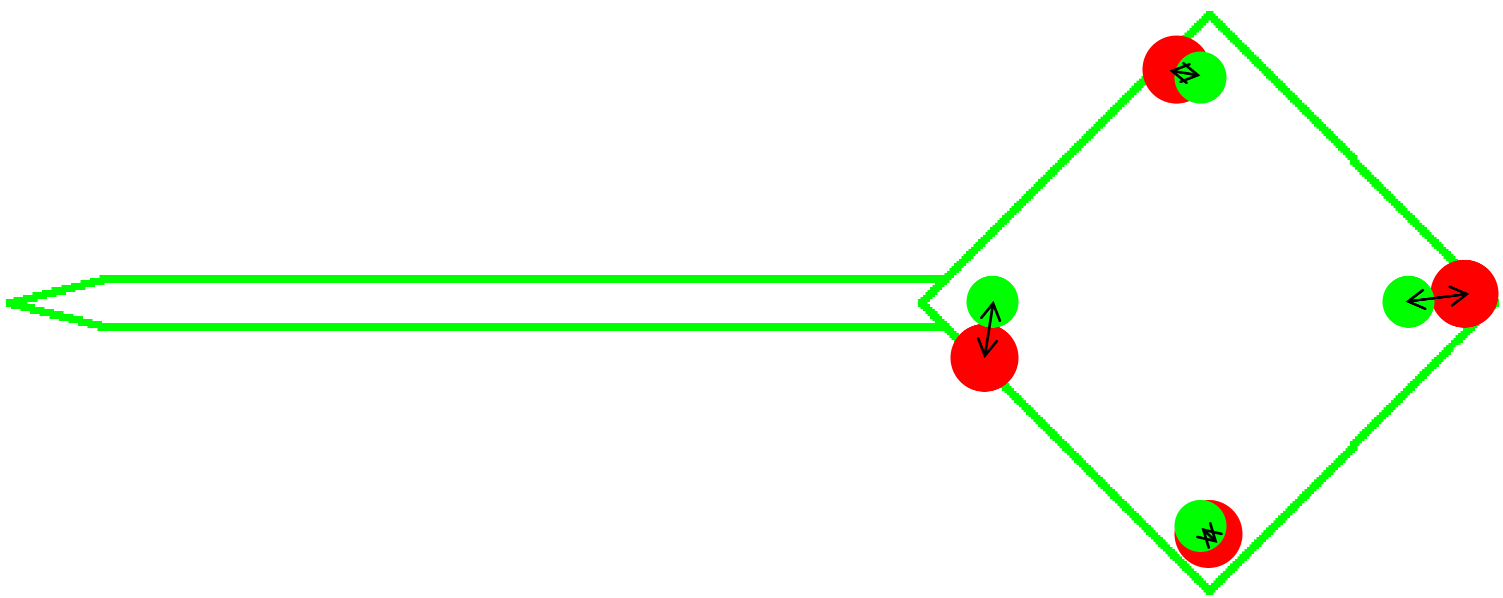
- similar problems appear in other research fields

Registration Error



Suppose we have a tracked pointing stylus with a DRB having 4 fiducial markers.

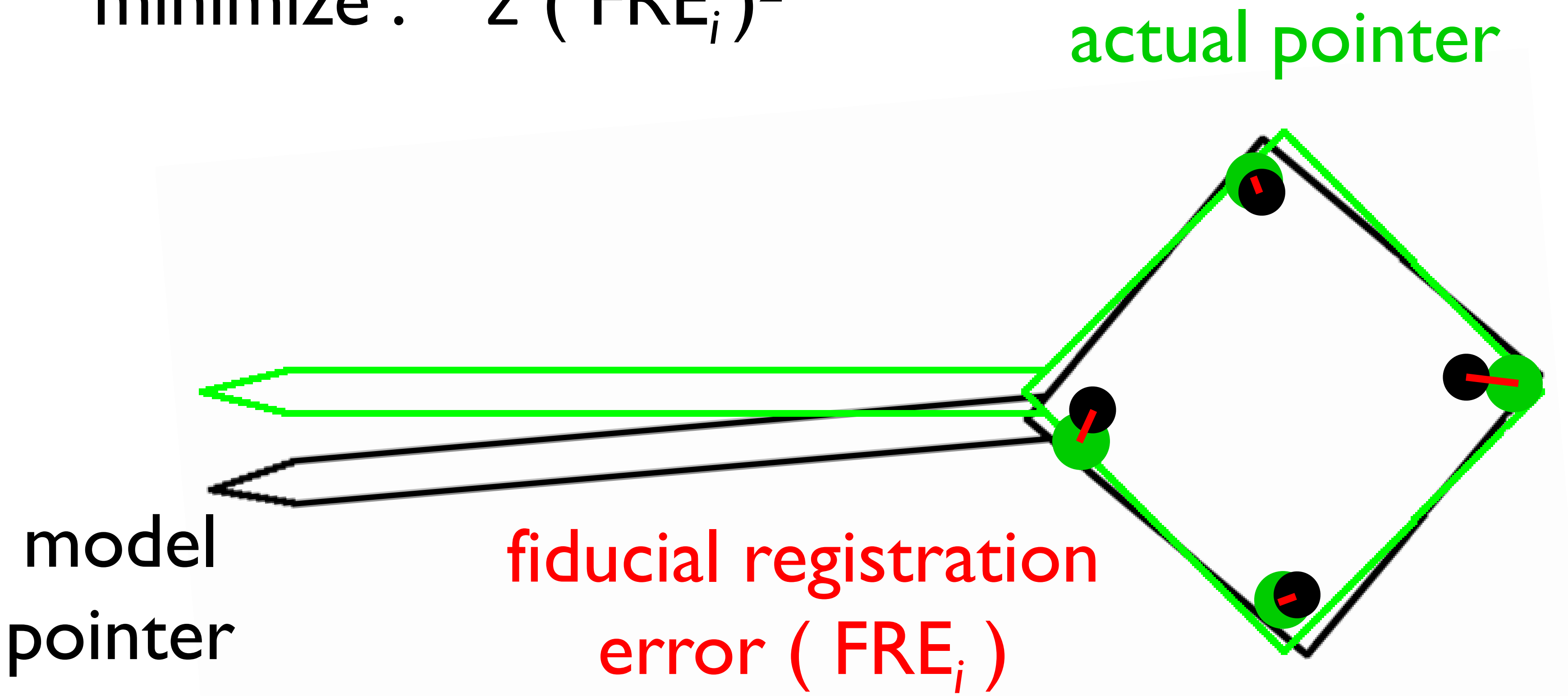
Registration Error



Because the of measurement errors in the tracking system, the locations of the fiducial markers cannot be measured exactly. The error between the actual and measured marker locations is called the fiducial localization error (FLE).

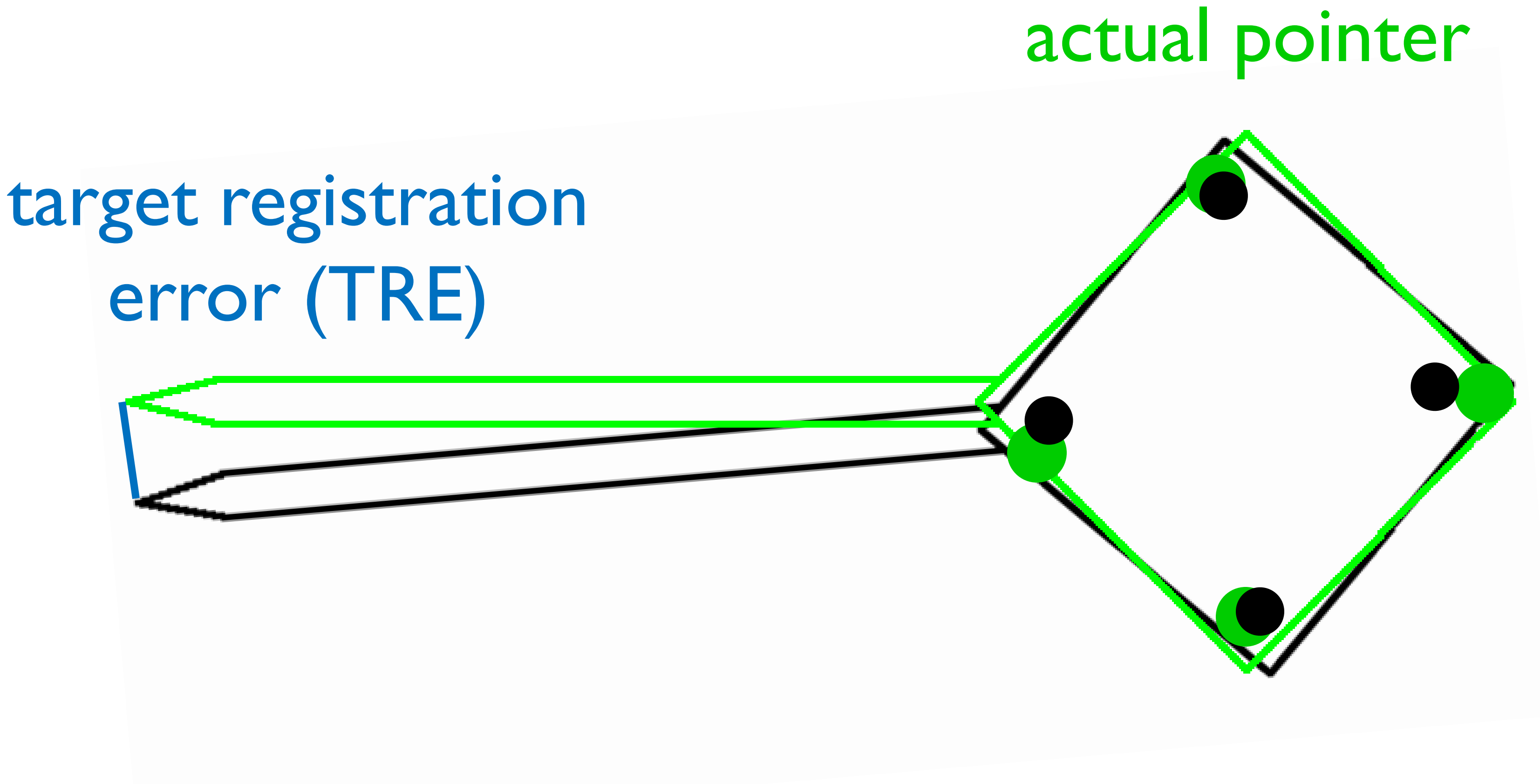
Registration Error

minimize : $\Sigma\ (\ FRE_i\ )^2$



When the model pointer is registered to the measured pointer, the FLE will lead to some error in the estimated rotation and translation. The residual errors in the fiducial locations after registration is called the fiducial registration error (FRE).

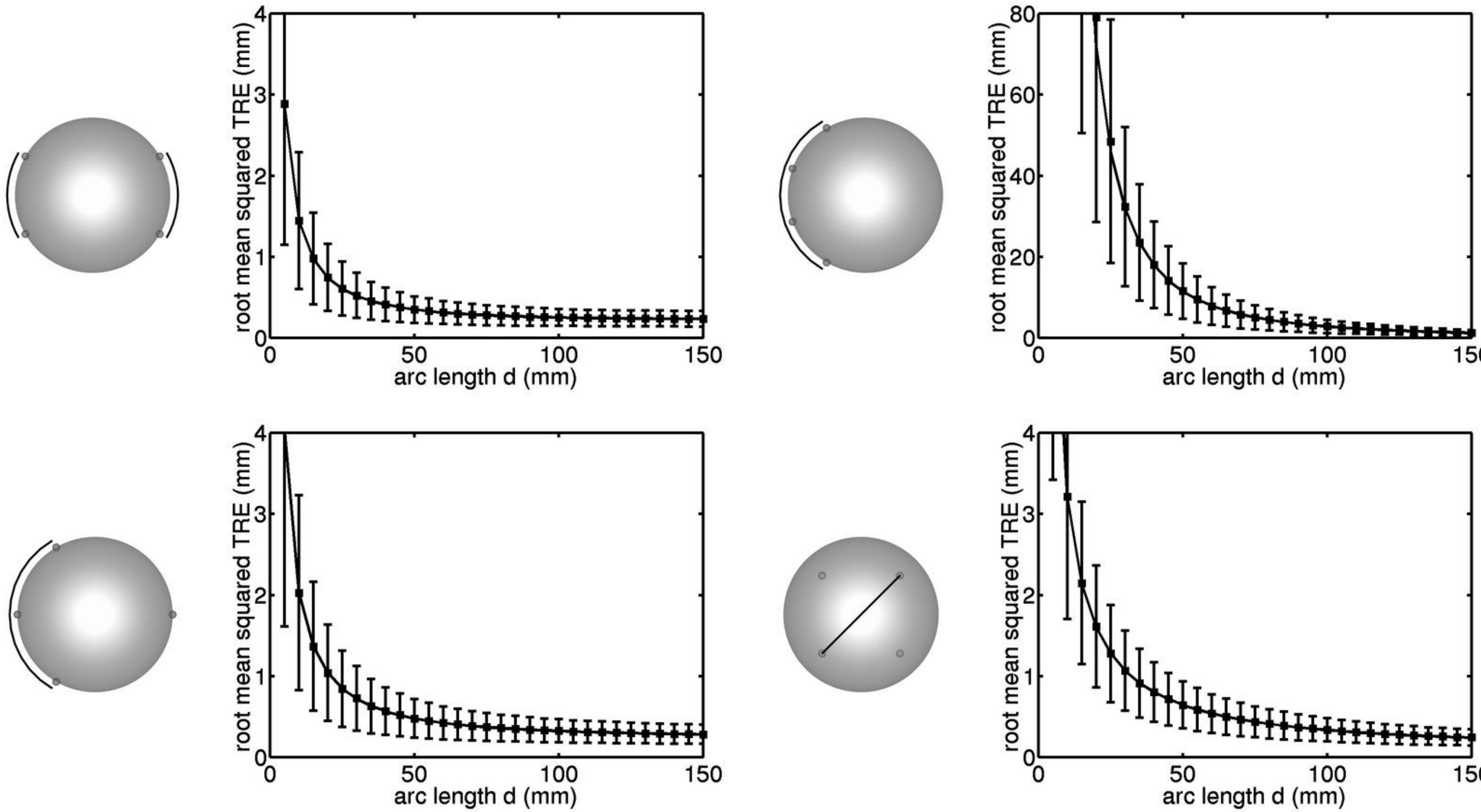
Registration Error



Usually, we are interested in points that are not fiducial locations. Any such point (not used for registration purposes) is called a target. The error between the true target position and registered target position is called the target registration error (TRE).

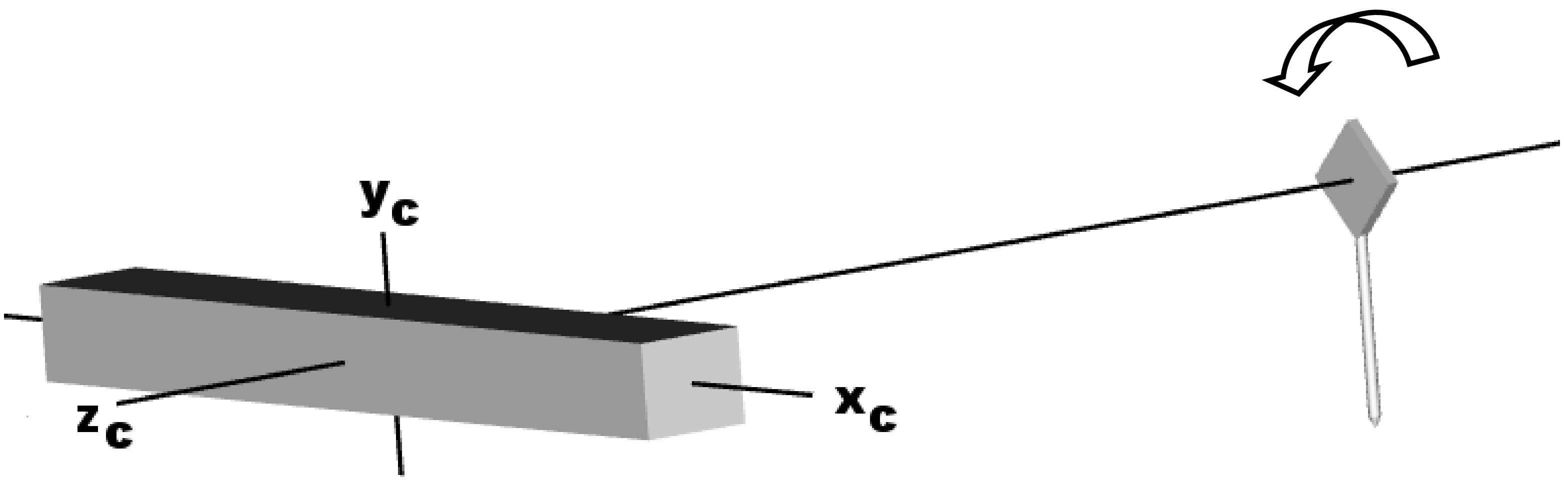
Analysis of TRE

- TRE for different configurations of markers



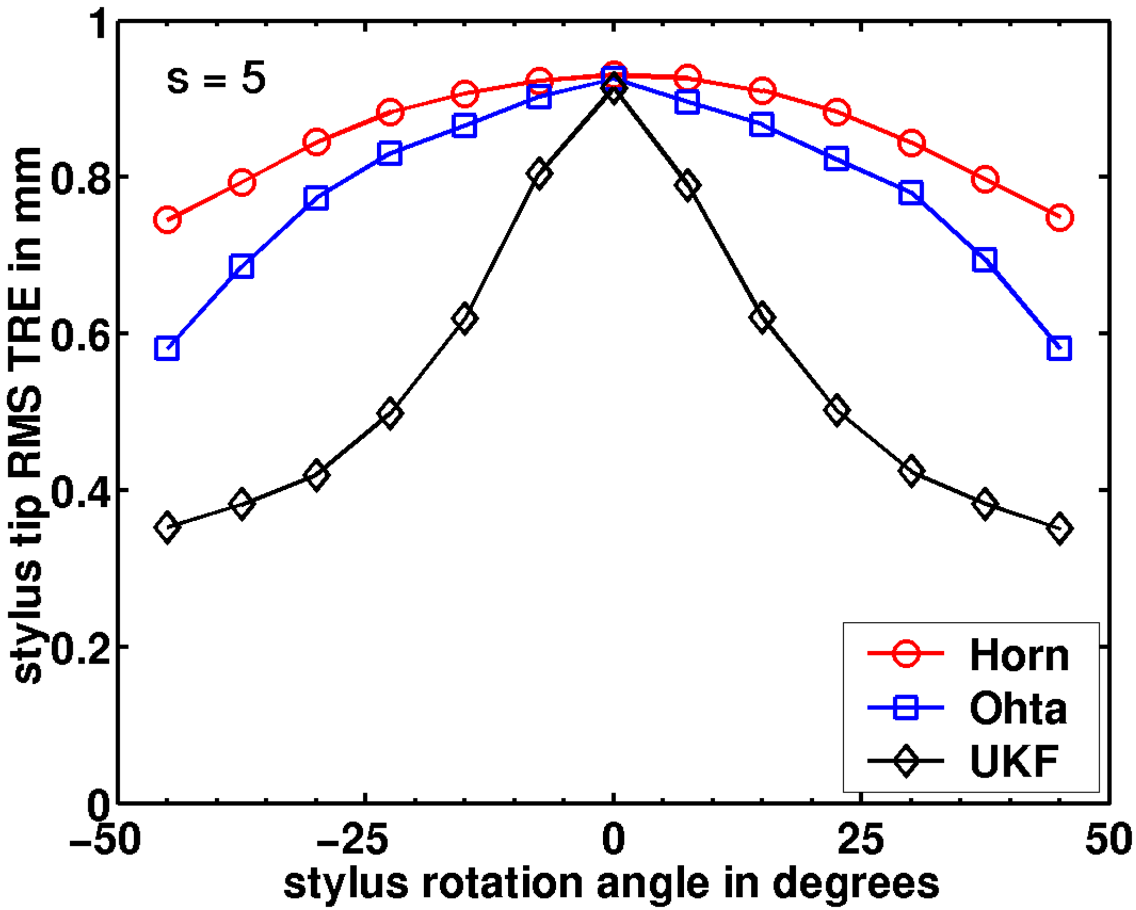
Anisotropic FLE

- what happens if the DRB rotates about x-axis?



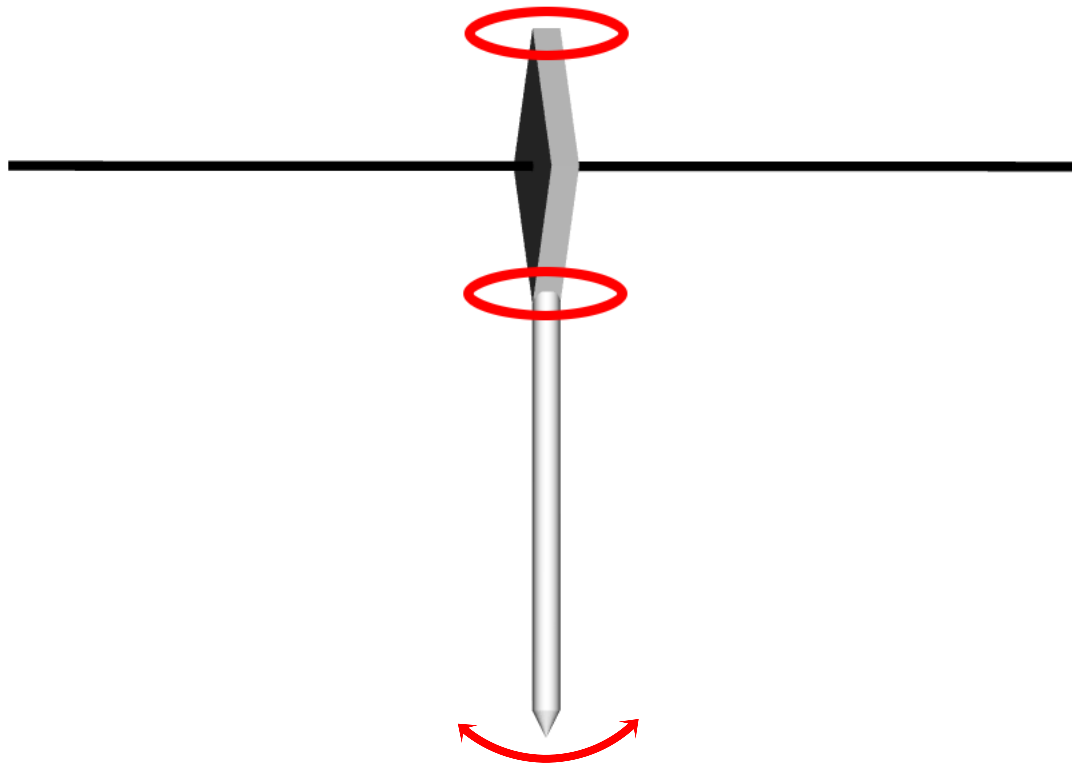
Anisotropic FLE

- TRE strongly dependent of rotation for anisotropic noise



Why the Peak in TRE?

- because rotational component of TRE_{RMS} is maximized when DRB faces the tracking camera



Why the Peak in TRE?

- and minimized when the DRB is perpendicular to the tracking camera

Observed TRE

- paradoxically, this behavior is exactly the opposite of what is observed in practice
  - TRE is typically worse when the DRB is rotated away from the camera